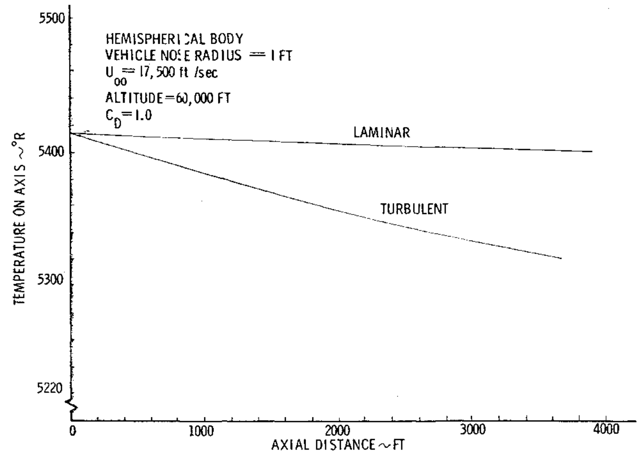

Fig. 2c Comparison of laminar and turbulent axial temperature distributions for an equilibrium wake.

The development of the combustion zone is clearly evident in the temperature profiles.

Figures 2a–2c show typical results for a fully laminar and fully turbulent axisymmetric wake. The program presently is being modified in an attempt to calculate boundary-layer flows with injection and gas-particle flows.

References

[1] Stoddard, F. J., "Finite-difference methods for computing viscous flows with arbitrary chemistry," Boeing Doc. D2-22406 (March 1963).

[2] Wu, J. C., "On the finite difference solution of laminar boundary layer problems," *Proceedings of 1962 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, Calif., 1962), pp. 55–69.

Influence of Wall Conductance on MHD Energy Conversion

William T. Snyder*

State University of New York at Stony Brook, Stony Brook, N. Y.

and

Lun King Liu†

Wayne State University, Detroit, Mich.

The purpose of this note is to examine the influence of finite electrical conductivity walls normal to the magnetic field in a constant area MHD generator configuration. Although the nonelectrode walls of an MHD generator are nominally insulators at room temperature, it is likely that, at the elevated operating temperatures, they will become slightly conducting.

In order to focus attention on the influence of the wall properties, certain simplifying assumptions will be made. The flow is assumed to be in the x direction, laminar, steady, with constant properties. A constant magnetic field is applied in the y direction, and the z dimension of the channel is large compared to the y dimension so that property changes in the z direction are negligible. The nonelectrode walls have thicknesses d_1 and d_2 and electrical conductivities σ_1 and σ_2.

Received June 14, 1963.

* Associate Professor of Engineering, Department of Thermal Sciences.

† Instructor, Department of Engineering Mechanics.

Hall effects are neglected so that simple Ohm's law is applicable.[1,2]

The equation of motion and the simplified form of Ohm's law appropriate to the present problem are

$$-\frac{dp}{dx} + \mu\frac{d^2u}{dy^2} - J_zB_y = 0 \qquad (1)$$

$$J_z = \sigma(E_z + uB_y) \qquad (2)$$

where E_z is constant and has the same value in the fluid and walls. It is convenient to introduce the following dimensionless parameters:

$$\eta = y/w \qquad \alpha = x/w \qquad \gamma = u/\bar{u}$$

$$\pi = \frac{p}{(\mu\bar{u}/w)} \qquad M = B_yw\left(\frac{\sigma}{\mu}\right)^{1/2} \qquad \Phi = \frac{E_z}{B_y\bar{u}} \qquad (3)$$

where the reference velocity $\bar{u}$ is the mean velocity defined as

$$\bar{u} = \frac{1}{2w}\int_{-w}^{w} udy \qquad (4)$$

The solution to Eq. (1) is the well-known Hartmann profile

$$\gamma = \left(\frac{M}{M - \tanh M}\right)\left(1 - \frac{\cosh M\eta}{\cosh M}\right) \qquad (5)$$

The total current per unit length in the x direction, including leakage current in the nonelectrode walls, is given by

$$I = \sigma_1\int_{-(w+d_1)}^{-w} E_zdy + \sigma\int_{-w}^{w}(E_z + uB_y)dy + \sigma_2\int_{w}^{w+d_2} E_zdy \qquad (6)$$

or

$$I^* = 2 + \Phi(2 + \theta_1 + \theta_2) \qquad (7)$$

where

$$I^* = \frac{I}{\sigma\bar{u}B_yw} \qquad \theta_1 = \frac{\sigma_1d_1}{\sigma w} \qquad \theta_2 = \frac{\sigma_2d_2}{\sigma w} \qquad (8)$$

The quantities θ_1 and θ_2 are the conductances of the lower and upper walls, respectively.

The quantity $2 + \theta_1 + \theta_2$ can be given a simple physical interpretation as follows. If L is the width of the channel in the z direction, then $R_i = L/2\sigma w$ represents the internal resistance of a fluid element of unit length in the x direction and of height $2w$. Likewise $R_1 = L/\sigma_1d_1$ and $R_2 = L/\sigma_2d_2$ are the resistances of the channel walls. A simple manipulation shows that

$$2 + \theta_1 + \theta_2 = 2(R_i/R^*) \qquad (9)$$

where

$$R^* = \left(\frac{1}{R_i} + \frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \qquad (10)$$

Thus R^* is the effective resistance of the three parallel resistances R_i, R_1, R_2, and $2 + \theta_1 + \theta_2$ is proportional to the ratio of fluid resistance to total resistance consisting of the parallel resistances of fluid and channel walls.

If V is the voltage difference across the channel in the z direction, then $V = -LE_z$. In MHD generator studies, it is convenient to work with a parameter expressing the ratio of operating voltage to open circuit voltage. Thus one defines

$$K = \frac{V}{V_{\text{open}}} = \frac{E_z}{E_z)_{\text{open}}} = \frac{\Phi}{\Phi_{\text{open}}} \qquad (11)$$

where Φ_{open} is obtained from Eq. (7) by setting $I^* = 0$. The power output per unit length in the x direction is given by the expression $\mathcal{P} = IV$. Substituting for I from Eqs. (7) and

(8) and introducing the voltage ratio from Eq. (11) gives

$$\mathcal{P} = 4\mu\bar{u}^2M^2\left(\frac{L}{w}\right)\frac{(K)(1-K)}{2+\theta_1+\theta_2} \tag{12}$$

Equation (12) shows the dependence of the power output on the wall conductances for fixed voltage ratio and mean velocity. For a given K and $\bar{u}$, the power is a maximum when $\theta_1 + \theta_2 = 0$, corresponding to perfectly insulating boundaries.

It is of interest to express the power output in terms of the pressure gradient. A relationship between mean velocity and pressure gradient can be obtained in the form

$$\bar{u} = \frac{(w^2/\mu)(dP/dx)}{[2M^2K/(2+\theta_1+\theta_2)] - [M^3/(M - \tanh M)]} \tag{13}$$

Combining Eqs. (12) and (13) gives

$$\mathcal{P} = \frac{4M^2(L/w)(K)(1-K)(w^4/\mu)(dP/dx)^2}{(2+\theta_1+\theta_2)\left(\dfrac{2M^2K}{2+\theta_1+\theta_2} - \dfrac{M^3}{M-\tanh M}\right)^2} \tag{14}$$

The voltage ratio that maximizes the power output can be obtained by setting the derivative of $\mathcal{P}$ with respect to K equal to zero. It is clear that the optimization can be done at either constant mean velocity or constant pressure gradient. If one of these parameters is held constant while the external loading conditions are changed, the other parameter must adjust itself in a manner compatible with the equation of motion. From Eq. (12), the condition

$$\partial\mathcal{P}/\partial K)_{\bar{u}} = 0$$

gives $K = \frac{1}{2}$ as the optimum voltage ratio at constant mean velocity. From Eq. (14), the condition

$$\partial\mathcal{P}/\partial K)_{dP/dx} = 0$$

gives

$$K = \frac{\frac{1}{2}}{1 + \{(\tanh M - M)/[M(2+\theta_1+\theta_2)]\}}$$

as the optimum voltage ratio at constant pressure gradient. There is thus a distinction between power optimization at constant mean velocity (or mass flow rate) and constant pressure gradient in that the latter case depends on the Hartmann number M and the wall conductances.

A final relationship of usefulness is that between the external load resistance R and the voltage ratio K. From Eq. (7), the total current per unit length of generator can be written as

$$I = 2\sigma\bar{u}B_yw(1-K) \tag{15}$$

The total current also is related to the external load through the relation

$$I = -\frac{LE_z}{R} = -\frac{LK\bar{u}B_y}{R}\Phi_{\text{open}} \tag{16}$$

Equating Eqs. (15) and (16) and substituting for Φ_{open} from Eq. (7) gives the results

$$R = \left(\frac{2R_i}{2+\theta_1+\theta_2}\right)\left(\frac{K}{1-K}\right) = R^*\left(\frac{K}{1-K}\right) \tag{17}$$

and

$$K = \frac{1}{1 + \{2R_i[R(2+\theta_1+\theta_2)]\}} = \frac{1}{1+(R^*/R)} \tag{18}$$

Equations (17) and (18) show the influence of the wall conductances on the relationship between external load and operating voltage ratio. Equating Eq. (18) to the values of K, which optimize the power output, then establishes the relationship between external load resistance and internal resistance of fluid and channel walls which must be satisfied for maximum power output. For constant mass flow rate, the ratio of internal to external resistance which maximizes the power output is

$$R^*/R = 1 \tag{19}$$

For constant pressure gradient operation, the corresponding ratio is

$$\frac{R^*}{R} = 1 + \frac{2(\tanh M - M)}{M(2+\theta_1+\theta_2)} \tag{20}$$

References

[1] Chang, C. C. and Lundgren, T. S., "The flow of an electrically conducting fluid through a duct with transverse magnetic field," *Proceedings Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, Calif., 1959), pp. 41–54.

[2] Chang, C. C. and Yen, J. T. "Magnetohydrodynamic channel flow as influenced by wall conductance," Z. Angew. Math. Phys. XIII, 266–272 (1962).

Transient Radiation Heating of a Rotating Cylindrical Shell

W. E. Olmstead,* L. A. Peralta,† and S. Raynor‡
Northwestern University, Evanston, Ill.

Nomenclature

r = radius of the cylinder, ft
s = wall thickness (assumed to be small compared with the radius), ft
k = thermal conductivity of the wall material, Btu/ft hr °R
σ = Stefan Boltzmann constant = 0.1717×10^{-8} Btu/ft^2 hr °R^4
c = specific heat of the wall material, Btu/lb °R
ρ = density of the wall material, lb/ft^3
a = average (with respect to wave length and angle of incidence) absorptivity of the wall material
e = average total hemispherical emissivity of the wall material for the spectrum of wave length radiated by the cylinder
K_s = energy received from the sun by a plane perpendicular to the "line of vision," Btu/ft^{2}hr
$K_N(t)$ = energy received from the thermal radiation pulse by a plane perpendicular to the "line of vision," Btu/ft^{2}hr
ψ = angular position fixed with respect to the rotation cylinder
θ = angular position fixed with respect to the heat sources

Introduction

A BRIEF survey of the literature pertaining to the heating of rotating shells by radiation can be found in Refs. 1 and 2. The problem considered here is that of a thin-walled circular cylinder, rotating with uniform velocity about its geometric axis. Initially, it has a temperature distribution corresponding to the equilibrium state with the solar radiation. It is then suddenly exposed to a time-dependent source of radiation. It is assumed that no heat exchange takes place inside of the cylinder and that external heat losses occur by thermal radiation only. Furthermore, it is assumed that the time-dependent source is on a "line of vision" between the cylinder and the sun (the equations easily can be

Received March 25, 1963; revision received July 8, 1963.
* Research Fellow, Department of Mechanical Engineering and Astronautical Sciences.
† Member of Technical Staff, The Mitre Corporation.
‡ Professor, Department of Mechanical Engineering and Astronautical Sciences.